

Research Highlights and Plans of the Lattice Hadron Physics Collaboration

John W. Negele

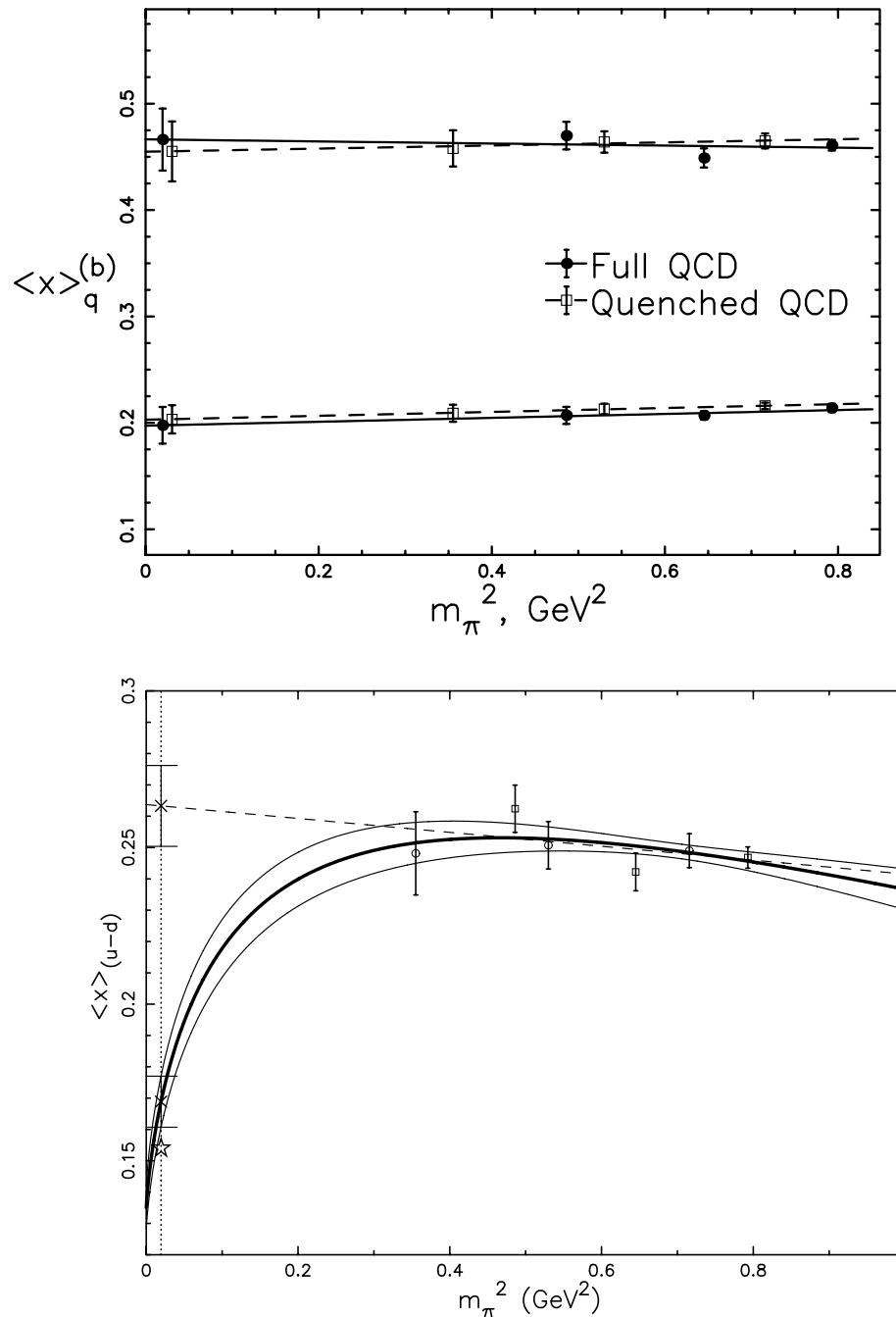
**LGT SciDAC Collaboration Meeting
February 21-22, 2003**

**Introduction
Spectroscopy
Hadron Structure
Algorithms
Plans**

Introduction

- Goal: Understand structure and interactions of hadrons from first principles
- Theoretical foundation for experimental program in hadron structure
 - Form Factors
 - Electromagnetic: nucleon, pion
 - Strangeness radius and moment:
HAPPEX, SAMPLE
 - Generalized Parton Distributions
 - Exotic Hadrons: Signature of gluonic excitations
- Previous effort
 - SESAM - Unquenched Wilson
 $m_\pi \geq 500 \text{ MeV}$

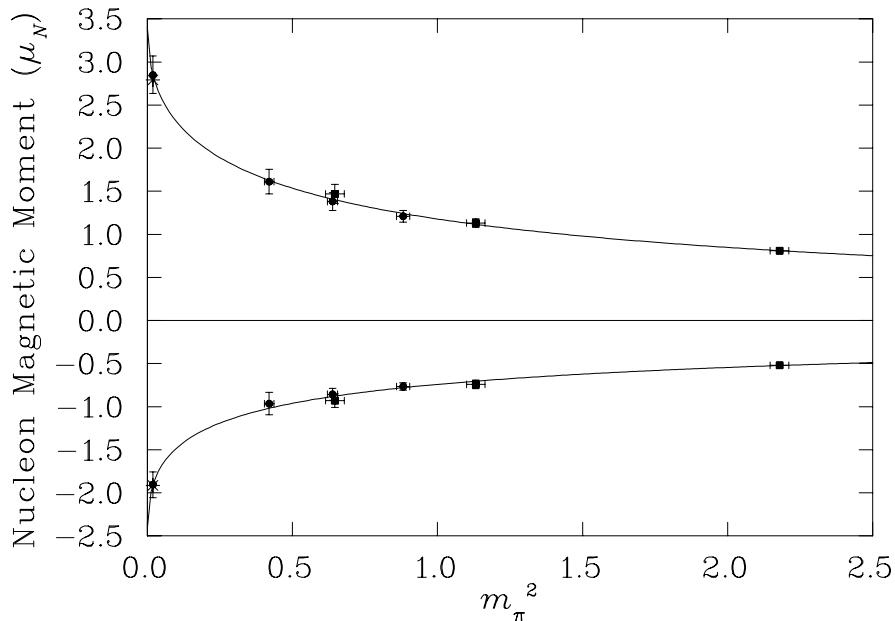
- Problem: extrapolation to light quark regime



Quark momentum fraction in the nucleon

D. Dolgov *et al.* hep-lat/0201021

- **Extrapolation to light quark regime**



Nucleon magnetic moment

D. Leinweber *et al.* hep-lat/0103006

- Recent focus: Techniques and QDP⁺⁺ code for spectroscopy, form factors, and generalized parton distributions
- Next phase: Hadronic physics at light quark masses - MILC sea and chiral valence quarks

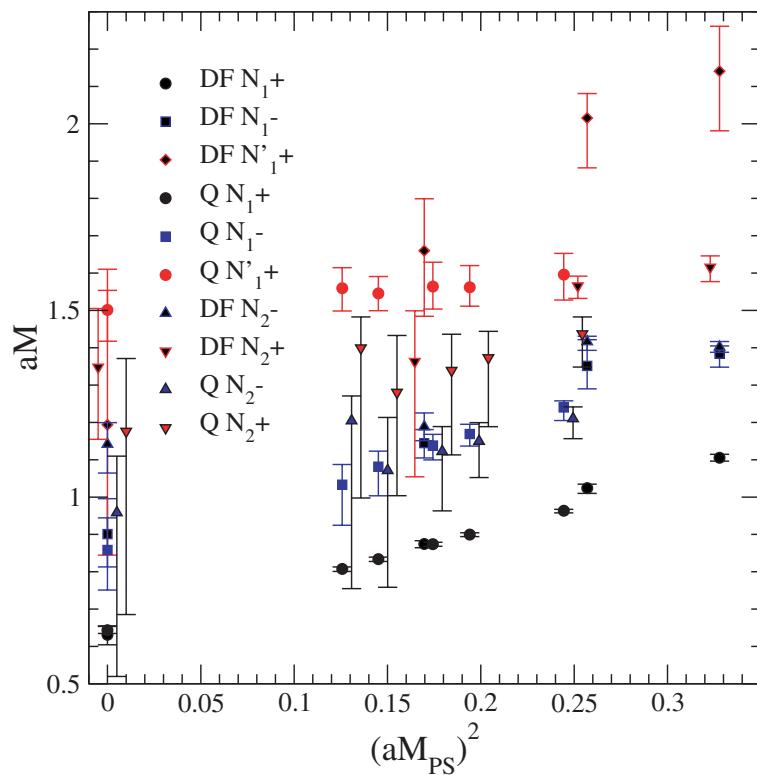
Spectroscopy

- Excitation of Nucleon Maynard and Richards
- Physics

Simple quark model Nature

$\frac{1}{2}^+$ Roper	$\frac{1}{2}^-$ Parity Partner
$\frac{1}{2}^-$ Parity Partner	$\frac{1}{2}^+$ Roper
$\frac{1}{2}^+$ Nucleon	$\frac{1}{2}^+$ Nucleon

- Understand ordering, structure
- Bayesian fitting of quenched and UKQCD dynamical improved Wilson fermions



Quenched Chiral Log and Roper Resonance

- Figure for $m_\pi^2 a^2 / ma$ on $16^3 \times 28$ Quenched Lattice with Overlap Fermion
- Iwasaki Gauge with $\beta = 2.265$, $a = 0.20$ fm from f_π
- $La \sim 3.2$ fm and Lowest $m_\pi = 183(7)$ MeV

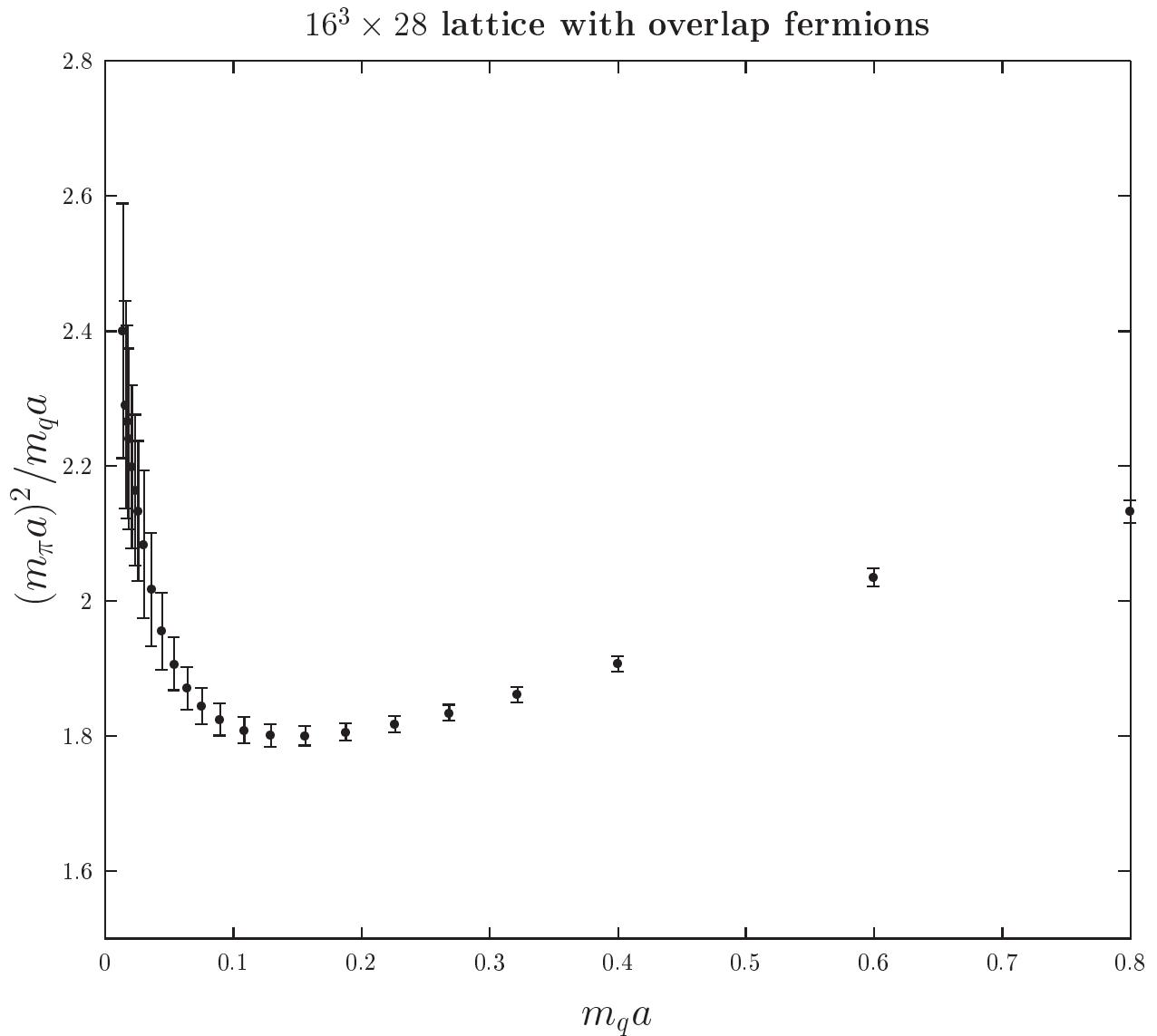


Figure 1: The ratio $m_\pi^2 a^2 / ma$ as a function of the quark mass ma .

Where Does One-loop Quenched Chiral Perturbation Apply?

$$m_\pi^2 = Am\{1 - \delta[\ln(Am/\Lambda_\chi^2) + 1]\} + A_\alpha m^2[1 + 2\ln(Am/\Lambda_\chi^2)] + Bm^2, \quad (1)$$

- Simultaneous fit of the parameters $A, \delta, \Lambda_\chi, A_\alpha, B$ with very weak constraints.
- Fit upwards from the fixed $m_{\pi min} = 180$ MeV and expand the range of $m_{\pi max}$ and plot as a function of $m_{\pi max}$.
- Criterion: The low-energy parameters remain constant in the range of m_π . Otherwise, $O(p^4)$ terms and second order perturbation are needed.
 - A and Λ_χ are fairly constant. The later is close to $4\pi f\pi$. Cannot draw much conclusion from A_α and B which are not important for $m_\pi < 300$ MeV.
 - Rapid transition in δ (\circ in Fig. 4) for $m_\pi \sim 300 - 400$ MeV. Fitting from $m_\pi \sim 900$ MeV downwards (\bullet in Fig. 4), as conventionally done, will not recover the high value of δ .

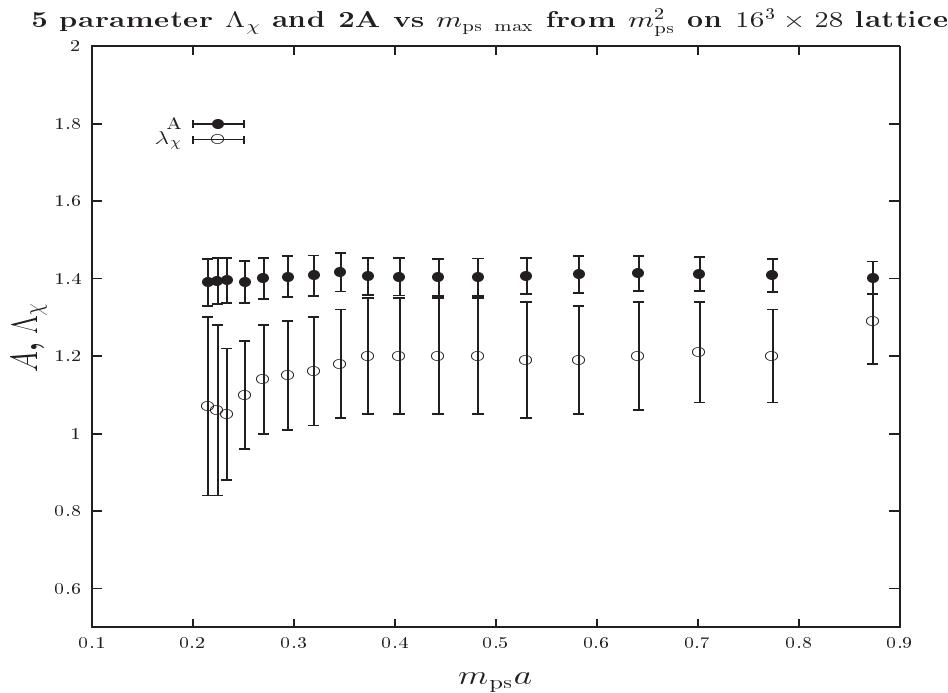


Figure 2: Fitted λ_χ and A as a function of the maximum pion mass squared. The minimum pion mass is at 183 MeV.

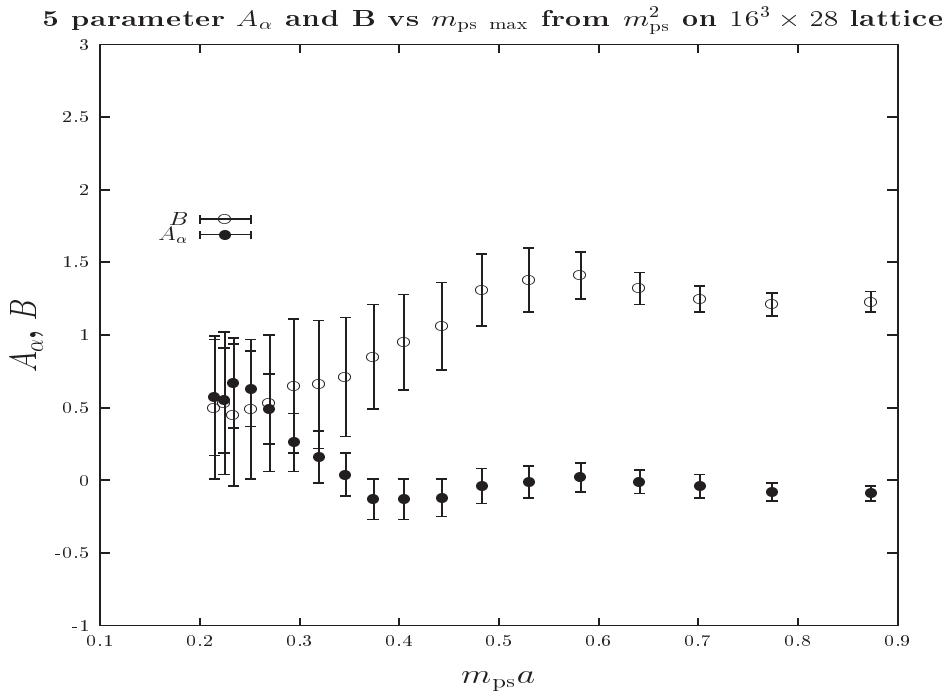


Figure 3: The same as the above figure for A_α and B as a function of the maximum pion mass squared.

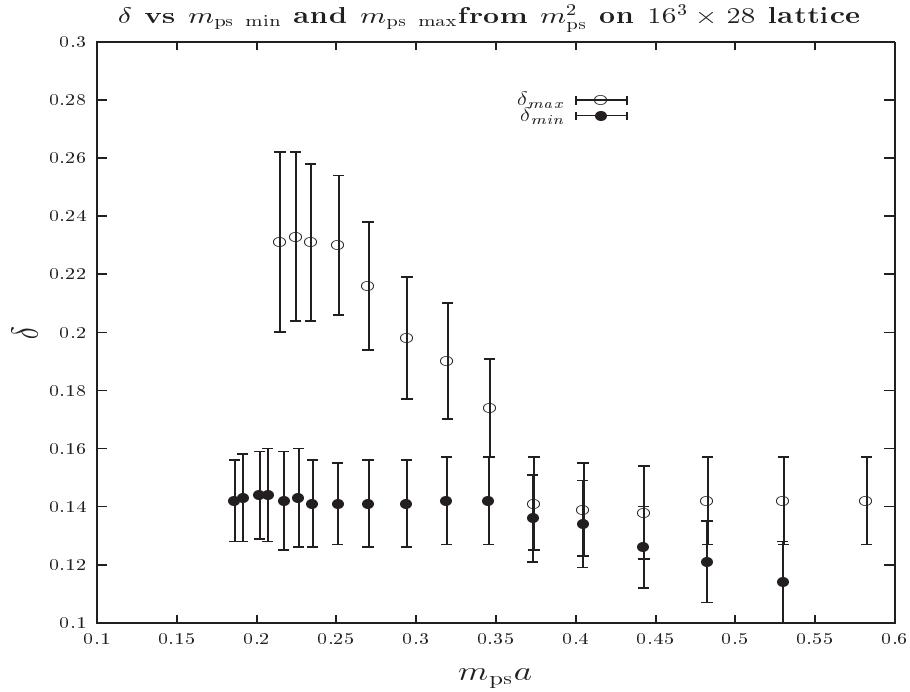


Figure 4: Quenched chiral δ as a function of the maximum pion mass squared. The minimum pion mass is at 183 MeV.

- Figure shows nucleon and Roper masses from a constraint fit which include Roper and the ghost $\eta'N$ state in the quenched approximation.
- The Roper has a fairly sharp drop in mass below $m_\pi \sim 400$ MeV and approaches the experimental result at 1440 MeV. This chiral behavior was not predicted before in models.
- The Roper was seen with the same interpolation field as the nucleon. Thus it is not a $q^4\bar{q}$ state nor an exotic hybrid state.

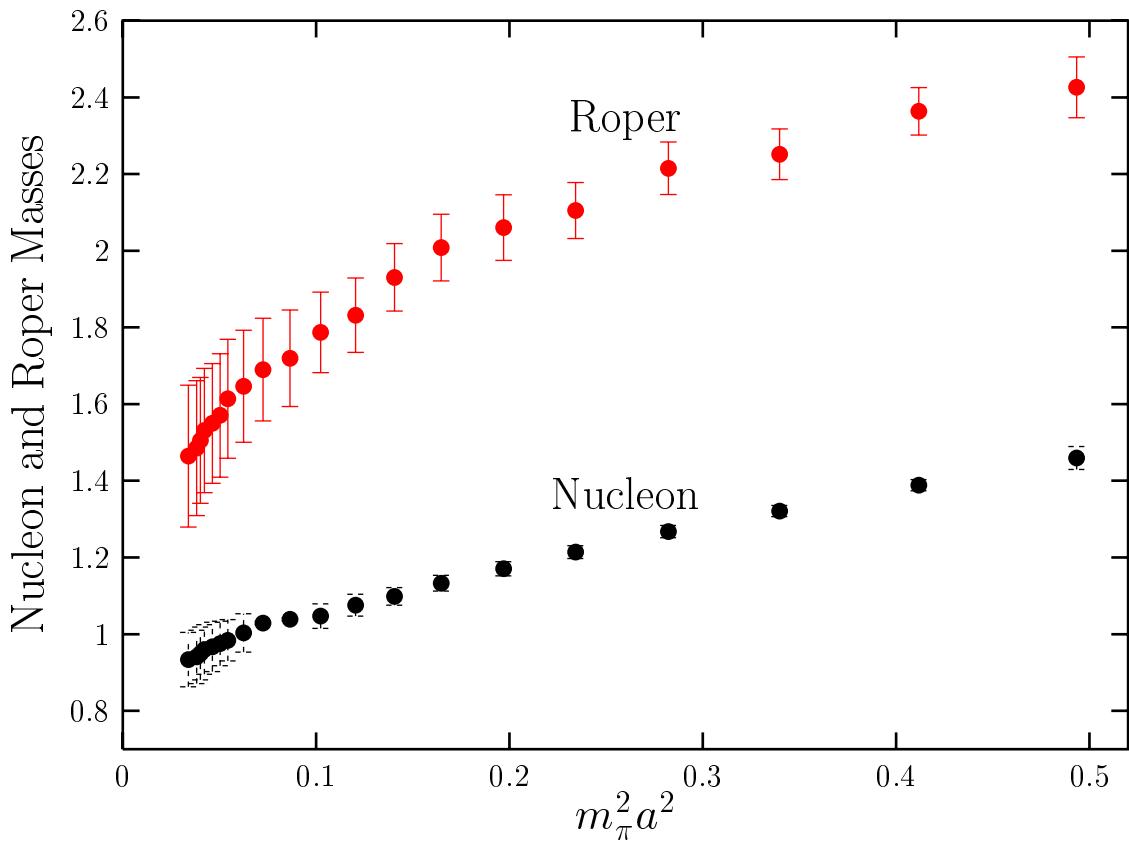


Figure 5: Nucleon and Roper masses from the constraint fit as a function of $m_\pi^2 a^2$.

- The ghost p-wave $\eta'N$ state and the Roper are nearly degenerate on the $16^3 \times 28$ lattice. This is possible with the weight of the $\eta'N$ state to the nucleon propagator constrained to be negative.
- The weight of the Roper and of the ghost $\eta'N$ state are plotted as a function of $m_\pi^2 a^2$. The $\eta'N$ state decouples above $m_\pi \sim 300$ MeV.

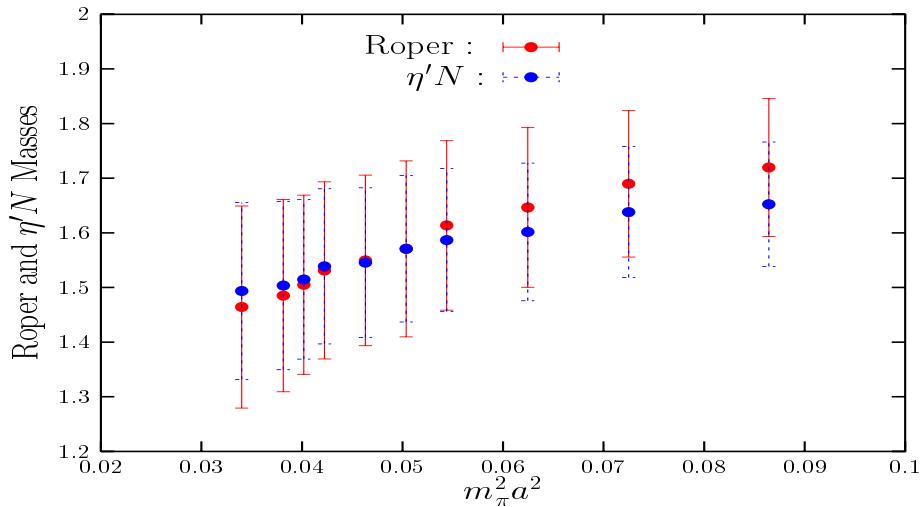


Figure 6: The Roper and p-wave $\eta'N$ mass as a function of $m_\pi^2 a^2$.

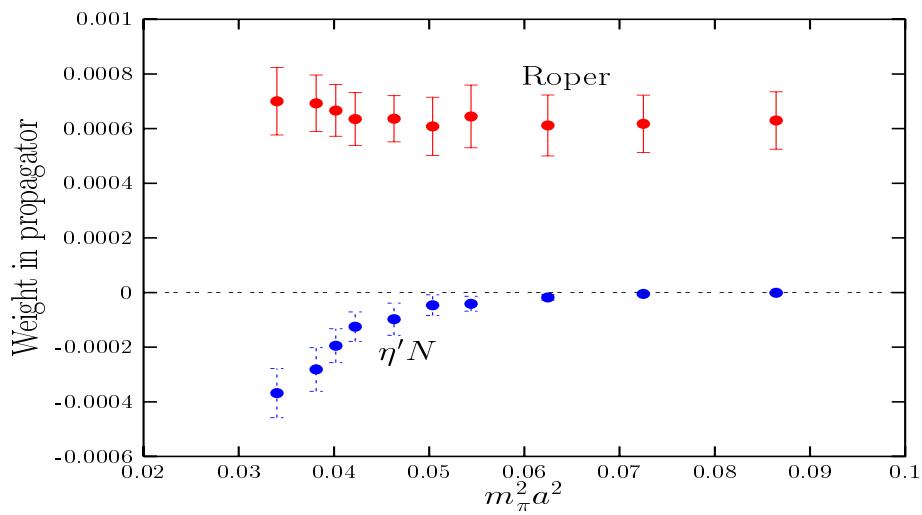


Figure 7: The weights of Roper and $\eta'N$ state to the nucleon propagator.

- Check on the volume dependence of $\eta'N$ and Roper: Since the $\eta'N$ is in p-wave, it should be sensitive to the volume with both η' and N having a momentum $k = 2\pi/La$. The interacting η and N are ~ 40 MeV above the free particle state (dashed lines). The Roper is fairly insensitive to the volume.

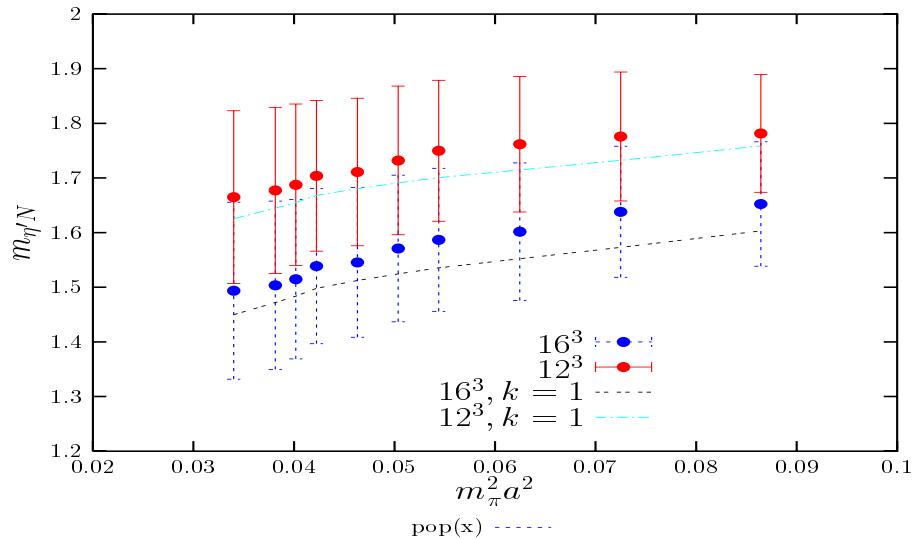


Figure 8: The $\eta'N$ mass on the $12^3 \times 28$ and $16^3 \times 28$ lattices. The dashed lines are the free $\eta'N$ state.

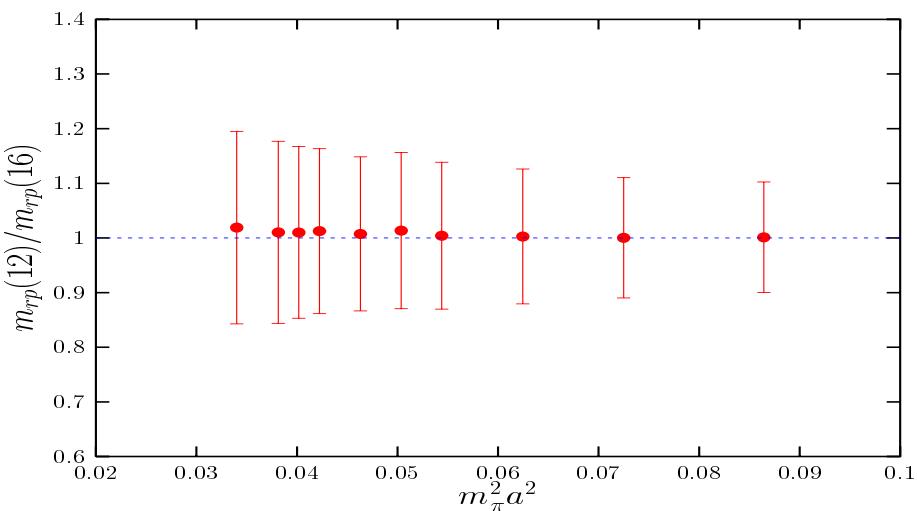
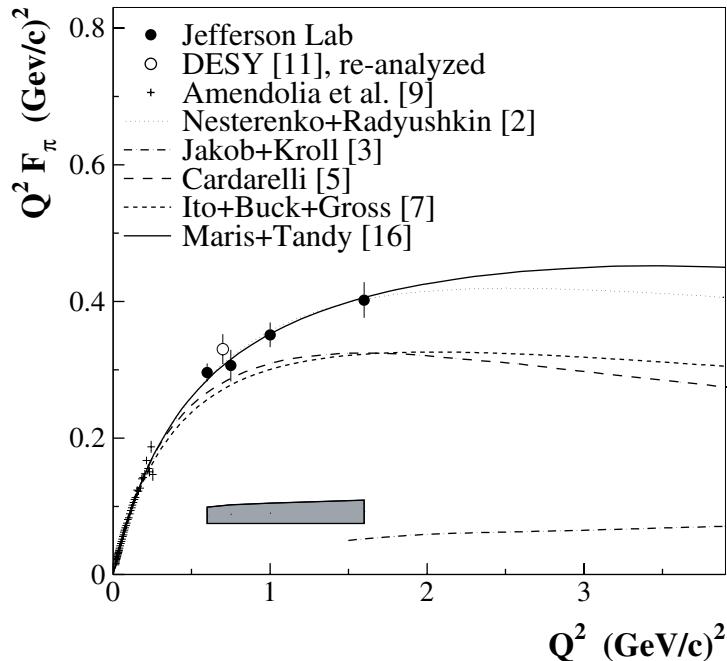


Figure 9: The ratio of Roper between $12^3 \times 28$ and $16^3 \times 28$ lattices.

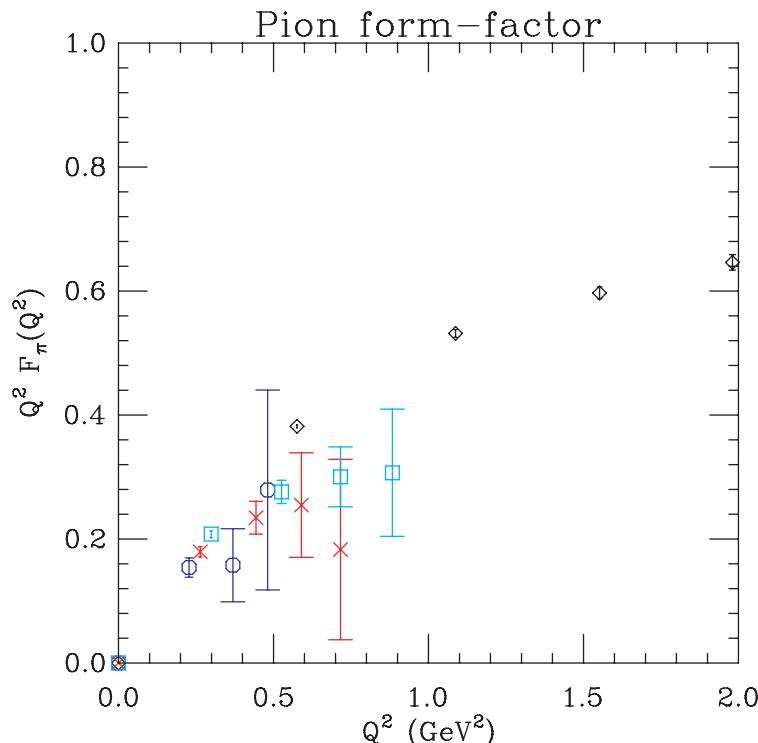
Hadron Structure

● Pion Form Factor

**Q^2 dependence - quark counting rules
New Jlab experiment → 2.5 GeV 2**



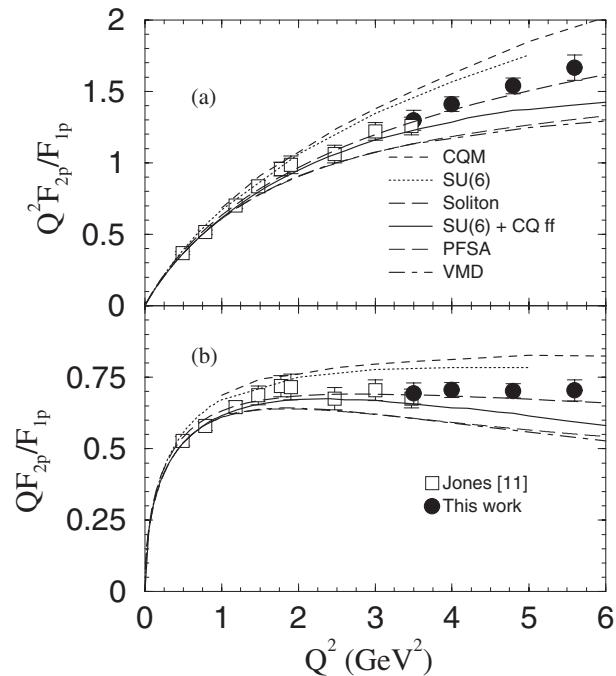
Exploratory calculation with domain wall fermions, quenched, $16^3 \times 32$ lattice Edwards et al.



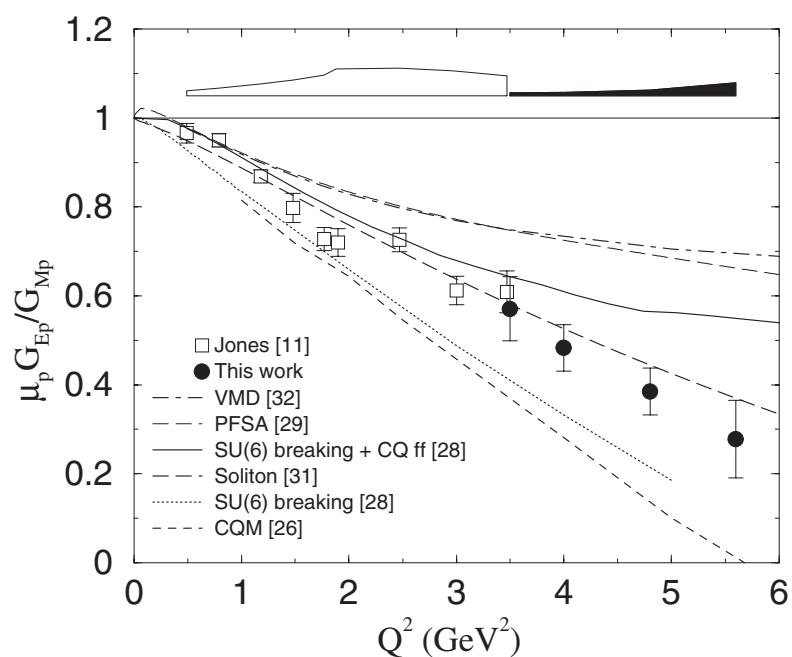
Nucleon Form Factor

- Precision experiments at Jlab

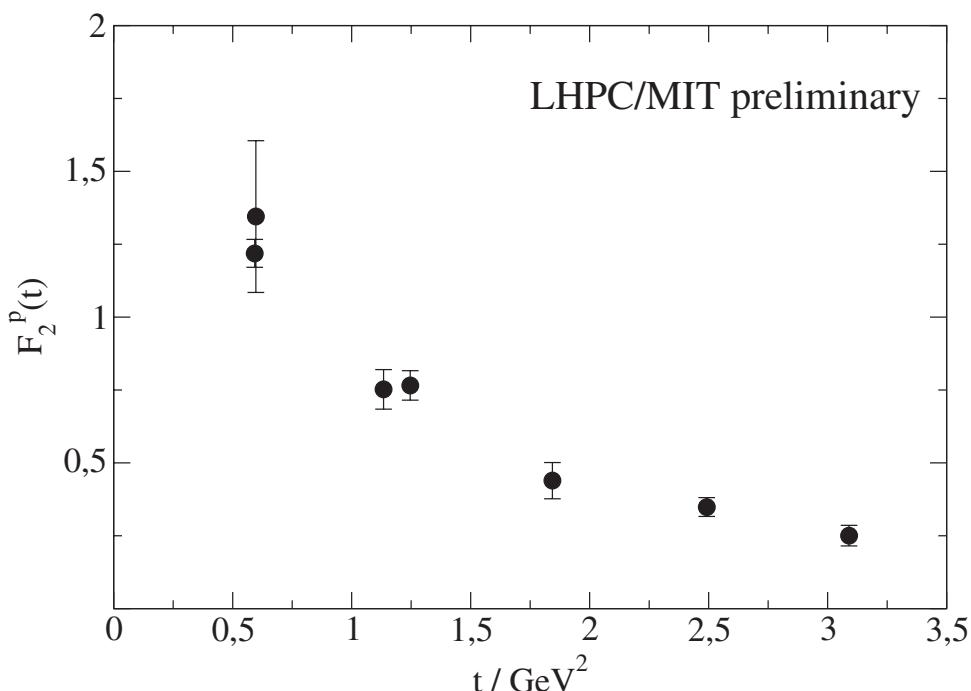
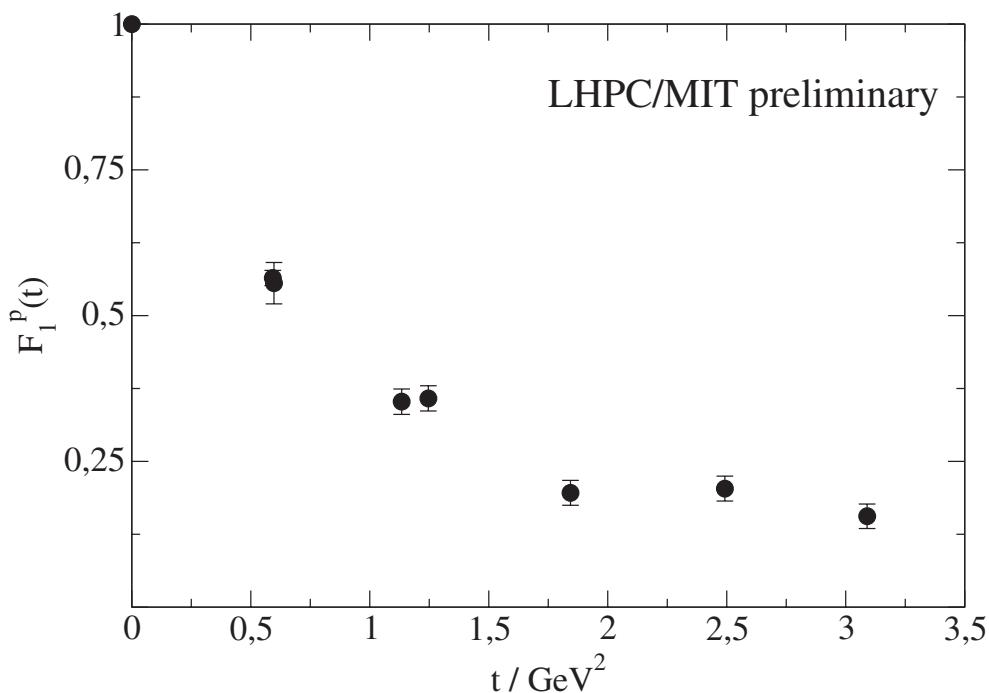
- Q^2 Dependence



- G_{EP}/G_{MP}



- Calculation with quenched and SESAM dynamical configurations
 - Reduction of statistical error
 - Breit frame
 - Multiple time sources
 - Fitting large-time 2- and 3- point functions
- Proton F_1 and F_2 Haegler, Renner, Schroers et al.



Generalized Parton Distributions

- Consider twist 2 operators

$$\mathcal{O}^{\mu_1 \dots \mu_n} = \bar{\psi} \gamma^{\{\mu_1} i D^{\mu_2} \dots i D^{\mu_n\}} \psi - \text{traces}$$

- Deep inelastic scattering

Forward M.E. → moment of quark distribution

$$\langle p | \mathcal{O}^{\mu_1 \dots \mu_n} | p \rangle \sim \int dx x^{n-1} q(x) p^{\mu_1} \dots p^{\mu_n}$$

- Deeply virtual Compton scattering

$$\langle p | \mathcal{O}^{\mu_1 \dots \mu_n} | p' \rangle$$

- Off-forward matrix elements

$$\bar{P} = \frac{1}{2}(P' + P), \quad \Delta = P' - P, \quad t = \Delta^2$$

$$\begin{aligned}\langle p | \mathcal{O}^{\mu_1} | p' \rangle &\sim \langle \gamma^{\mu_1} \rangle A_{10}(t) \\ &\quad + \langle \sigma^{\mu_1 \alpha} \rangle \Delta_\alpha B_{10}(t)\end{aligned}$$

$$\begin{aligned}\langle p | \mathcal{O}^{\mu_1 \mu_2} | p' \rangle &\sim \langle \gamma^{\{\mu_1} \bar{P}^{\mu_2\}} A_{20}(t) \\ &\quad + \langle \sigma^{\{\mu_1 \alpha} \Delta_\alpha \bar{P}^{\mu_2\}} B_{20}(t) \\ &\quad + \Delta^{\{\mu_1 \Delta^{\mu_2\}} C_{20}(t)\end{aligned}$$

$$\begin{aligned}\langle p | \mathcal{O}^{\mu_1 \mu_2 \mu_3} | p' \rangle &\sim \langle \gamma^{\{\mu_1} \bar{P}^{\mu_2} \bar{P}^{\mu_3\}} A_{30}(t) \\ &\quad + \langle \sigma^{\{\mu_1 \alpha} \Delta_\alpha \bar{P}^{\mu_2} \bar{P}^{\mu_3\}} B_{30}(t) \\ &\quad + \langle \gamma^{\{\mu_1} \Delta^{\mu_2} \Delta^{\mu_3\}} A_{31}(t) \\ &\quad + \langle \sigma^{\{\mu_1 \alpha} \Delta_\alpha \Delta^{\mu_2} \Delta^{\mu_3\}} B_{31}(t)\end{aligned}$$

- Limits

- Moments of parton distributions $t \rightarrow 0$

$$A_{n0}(0) = \int dx x^{n-1} q(x)$$

- Form factors

$$A_{10}(t) = F_1(t) \quad B_{10}(t) = F_2(t)$$

- Total quark angular momentum

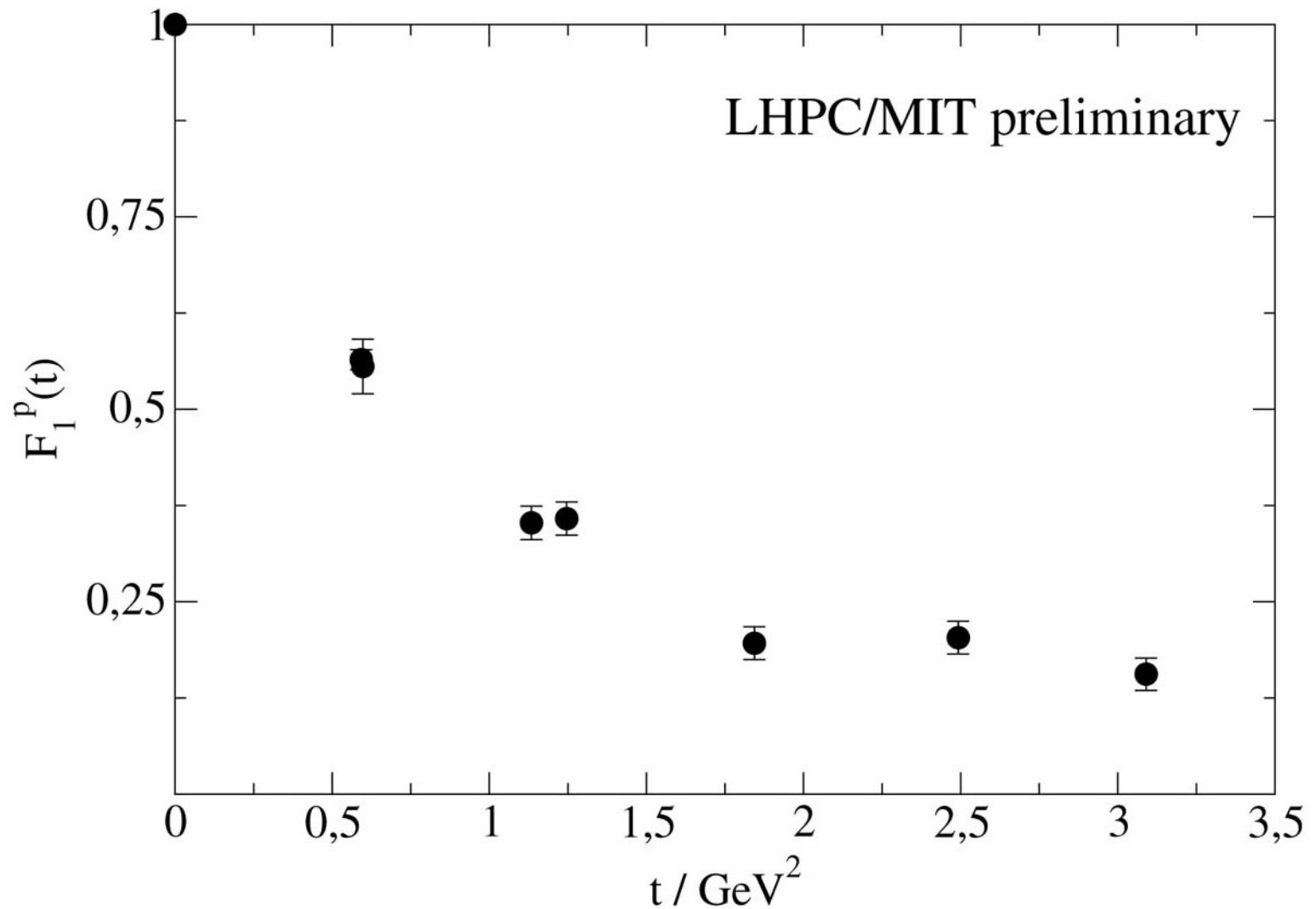
$$J_q = \frac{1}{2}[A_{20}(0) + B_{20}(0)]$$

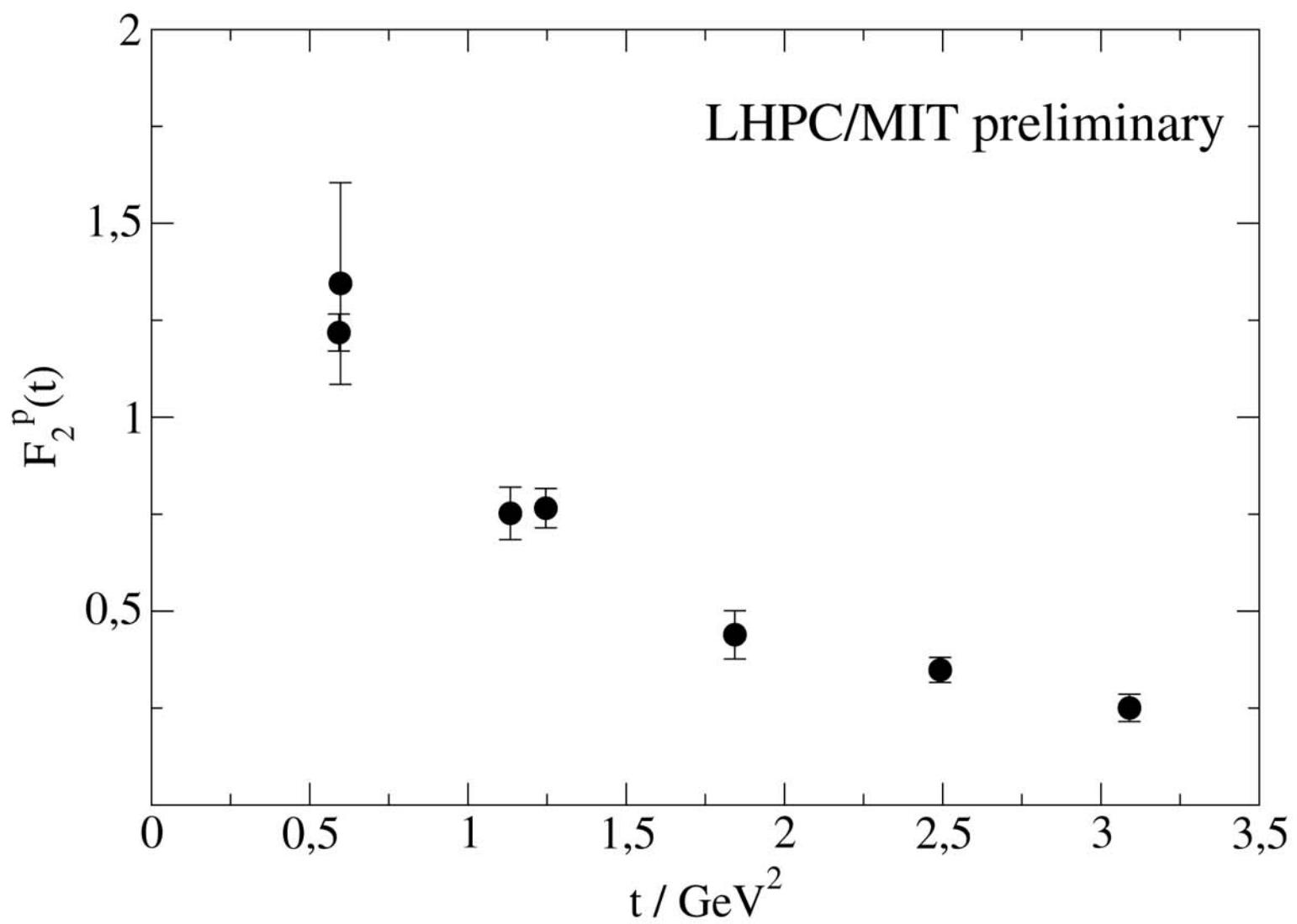
- t-Dependence

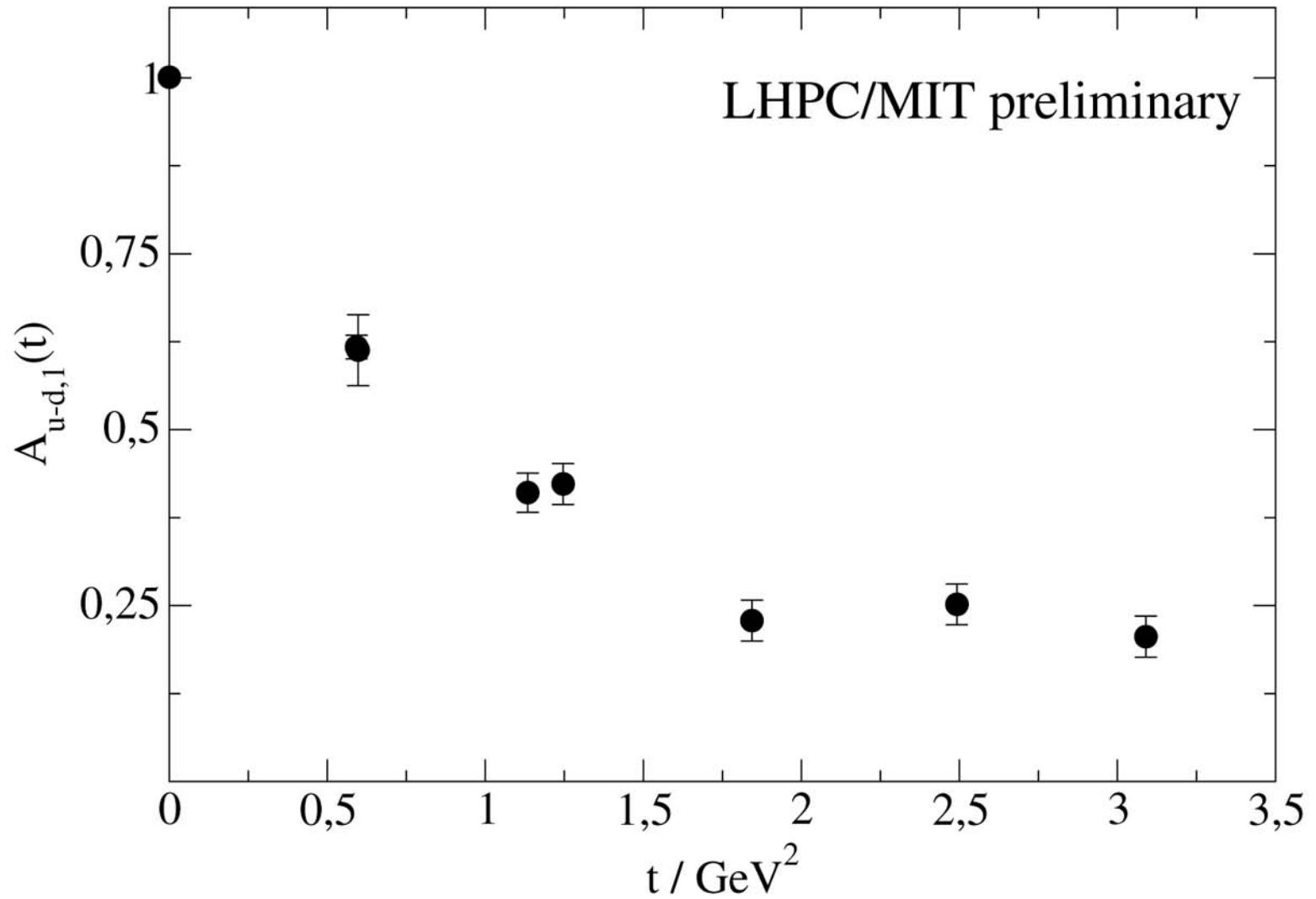
$q(x, t)$: Transverse Fourier transform of light cone parton distribution at given x

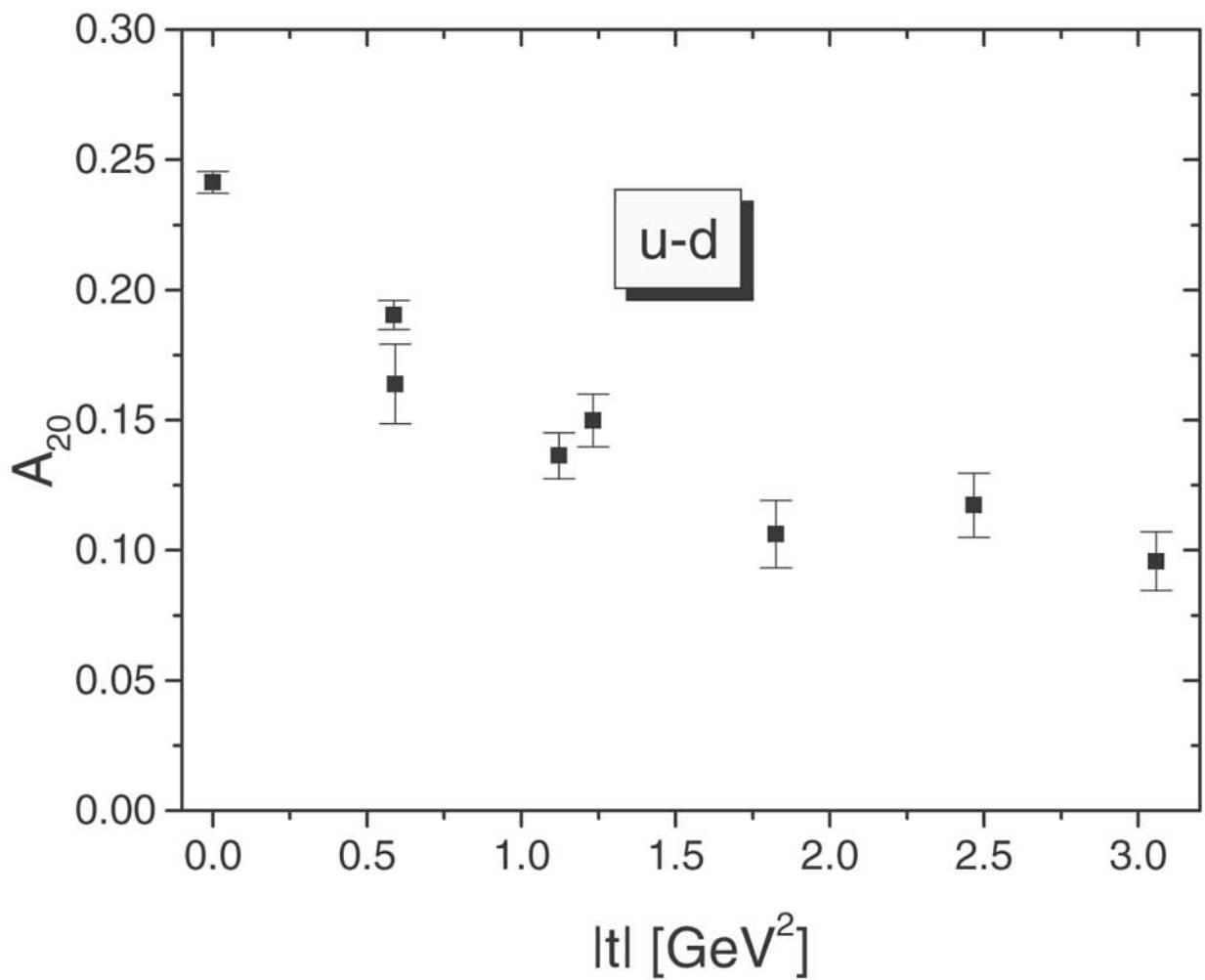
$x \rightarrow 1$: Single Fock space component - slope $\rightarrow 0$.

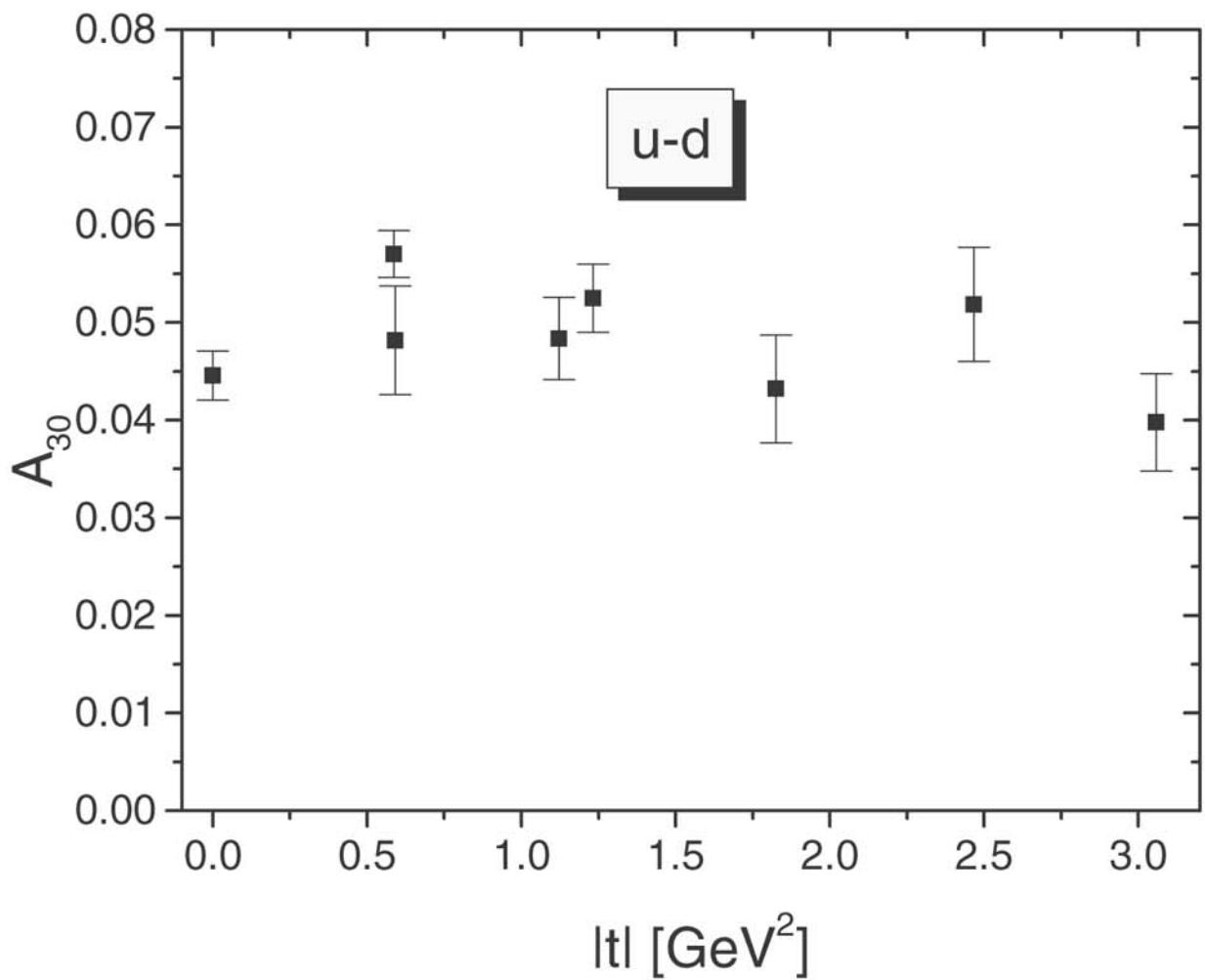
$x < 1$: Transverse structure- slope steeper

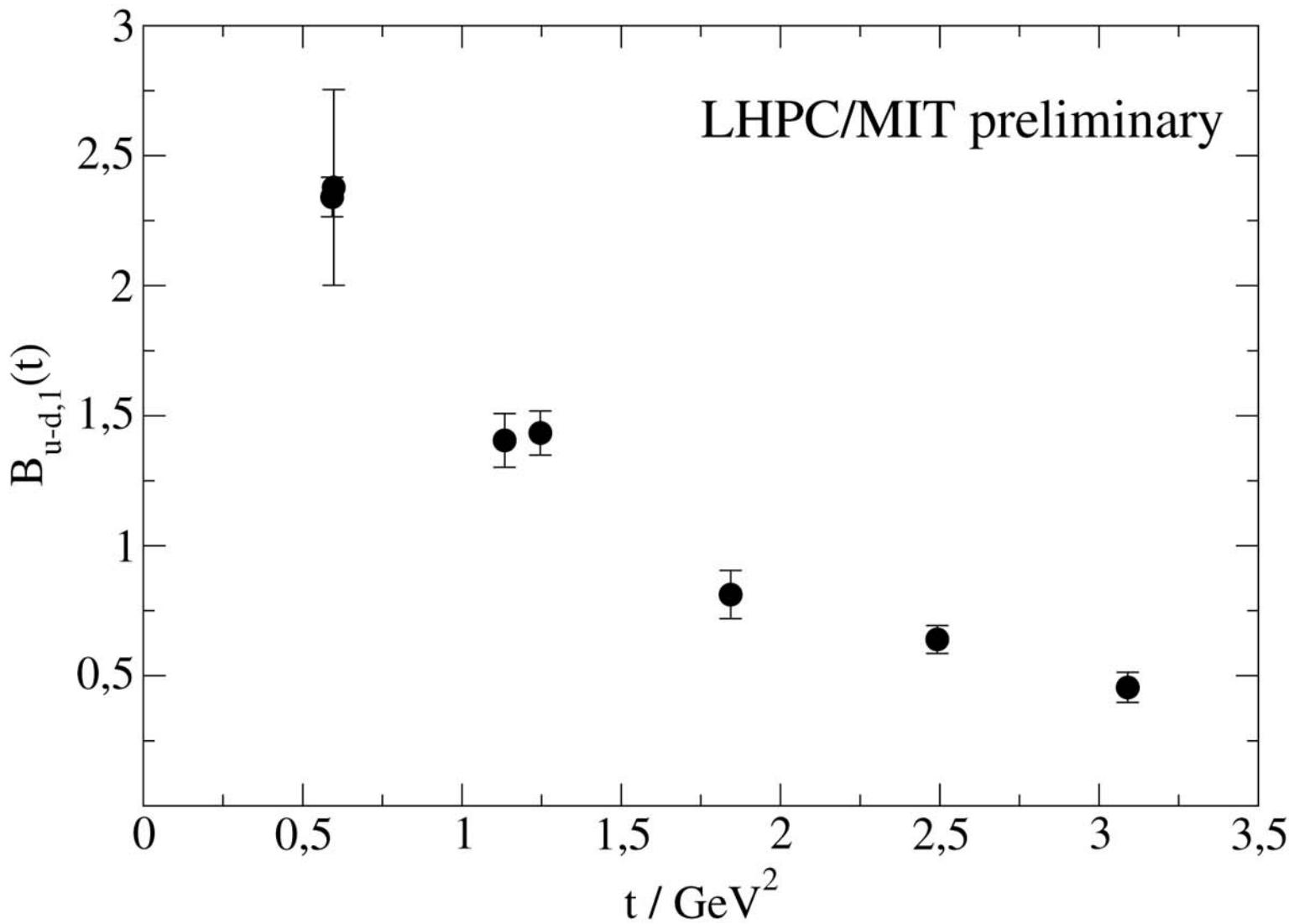


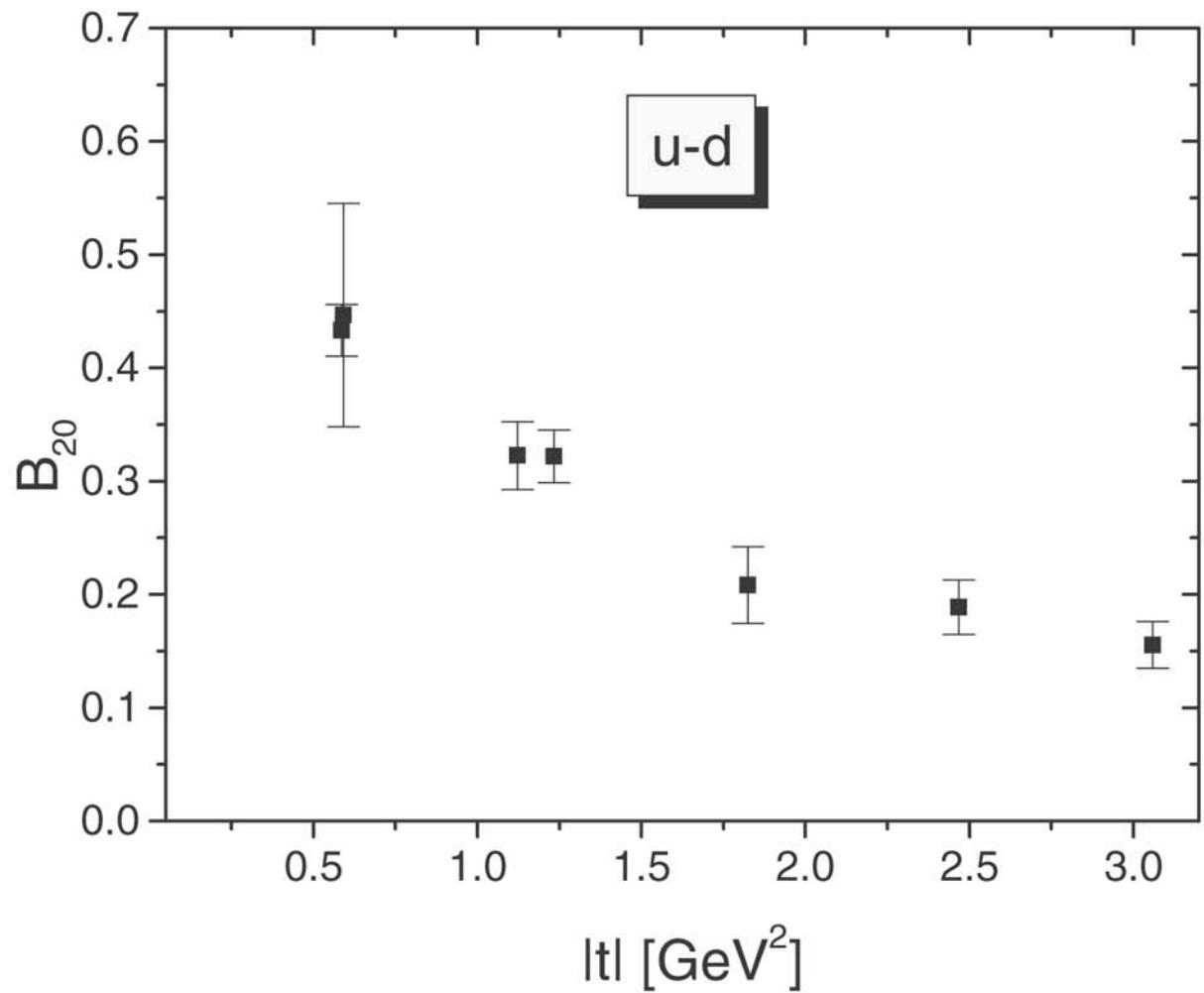


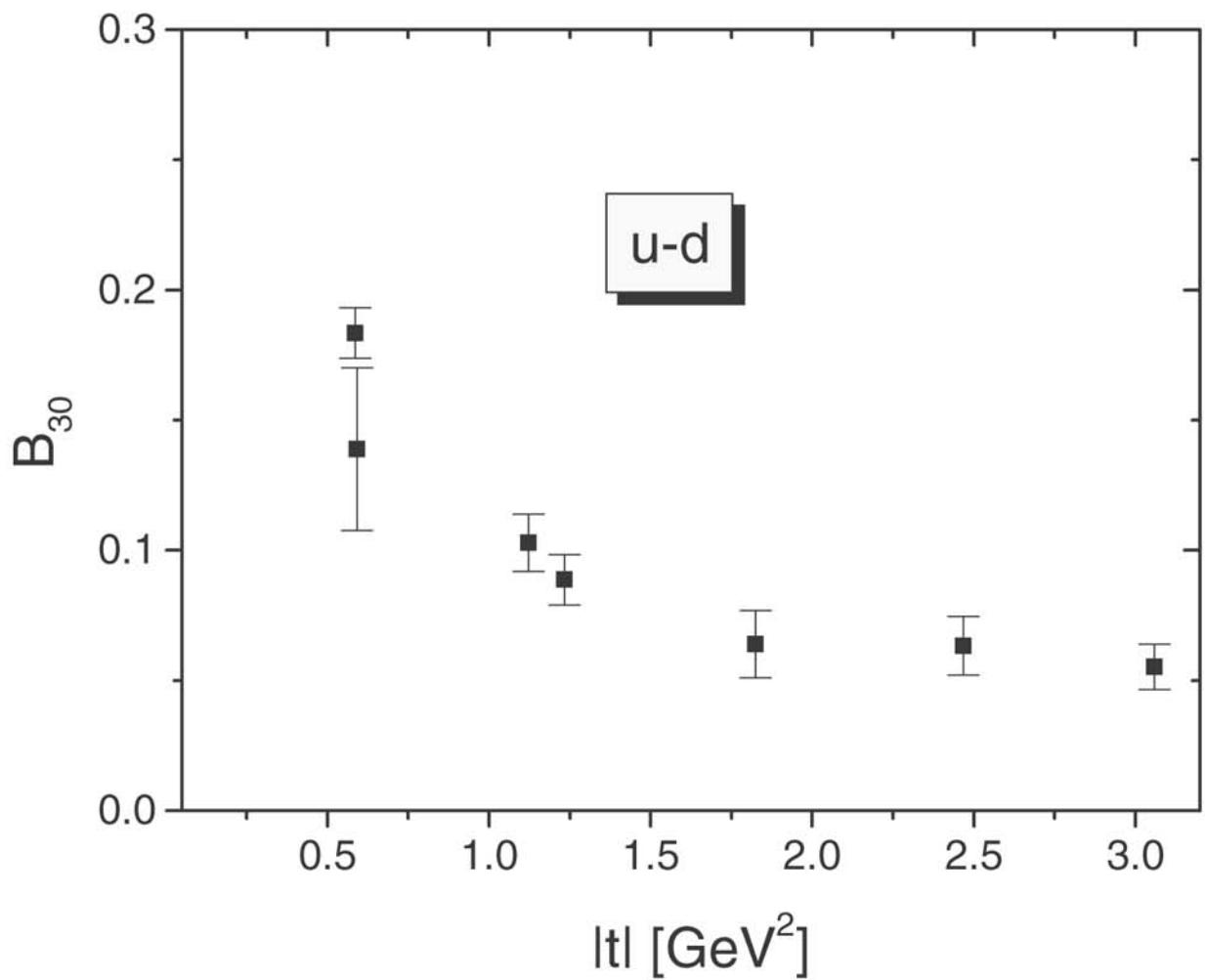


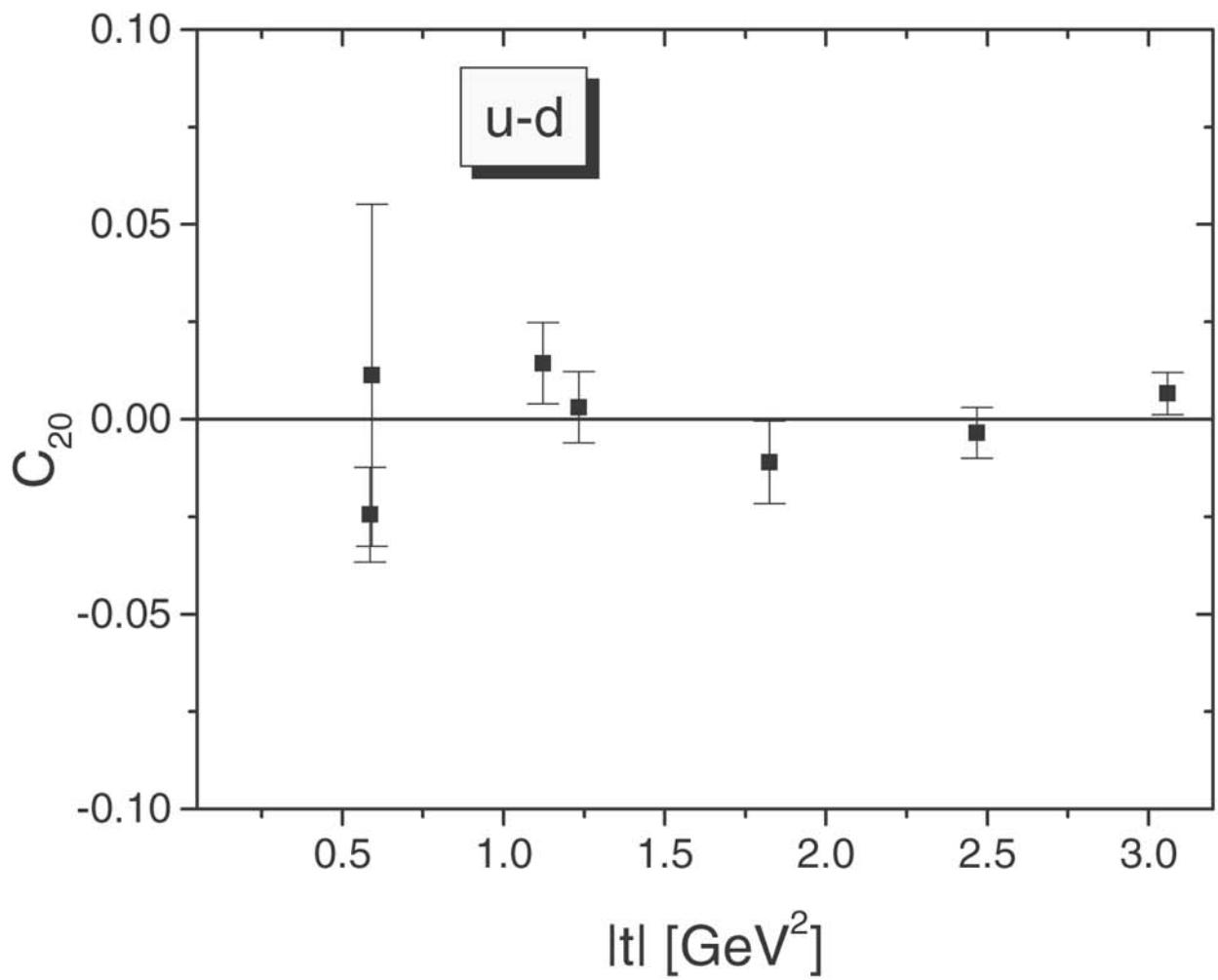


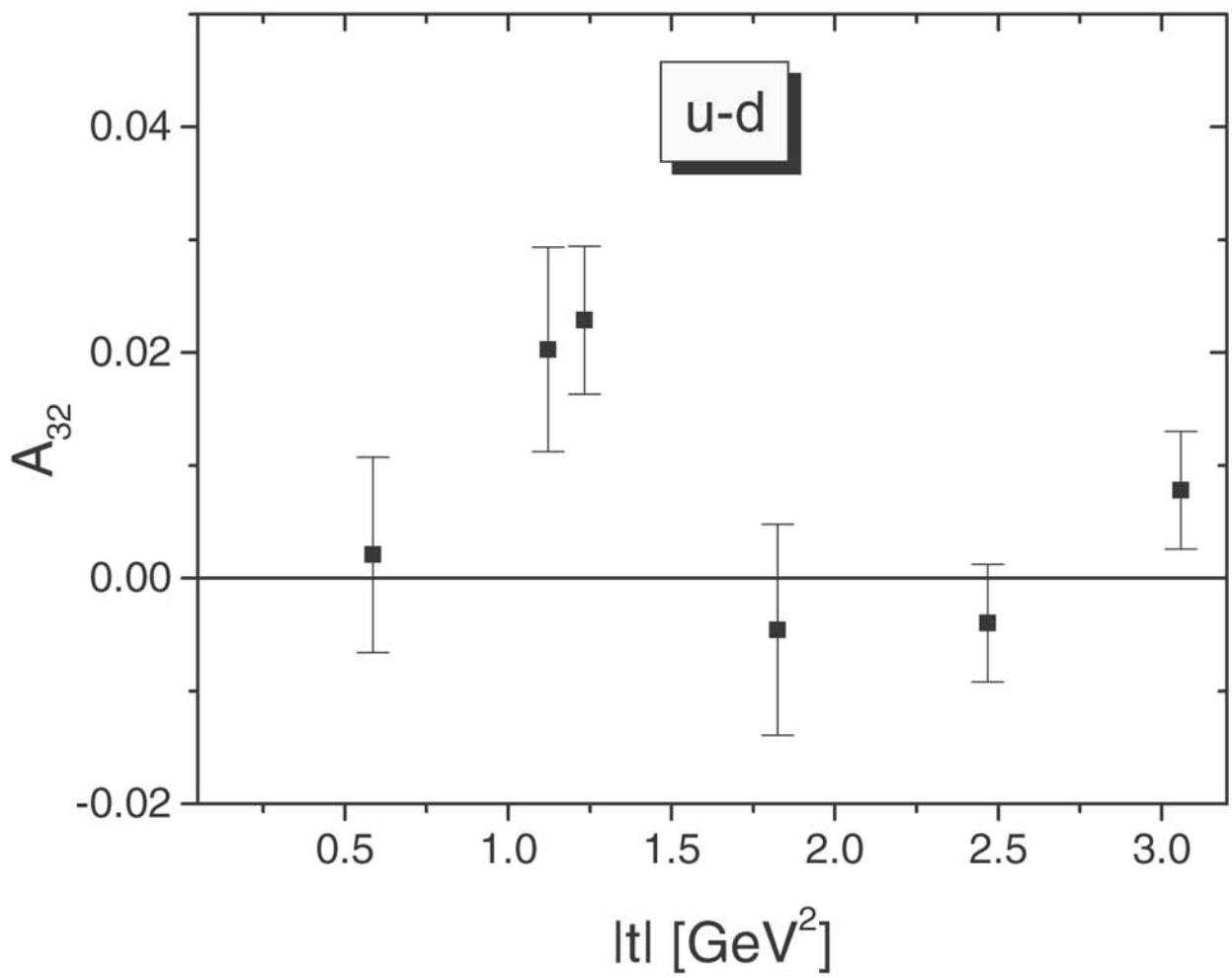


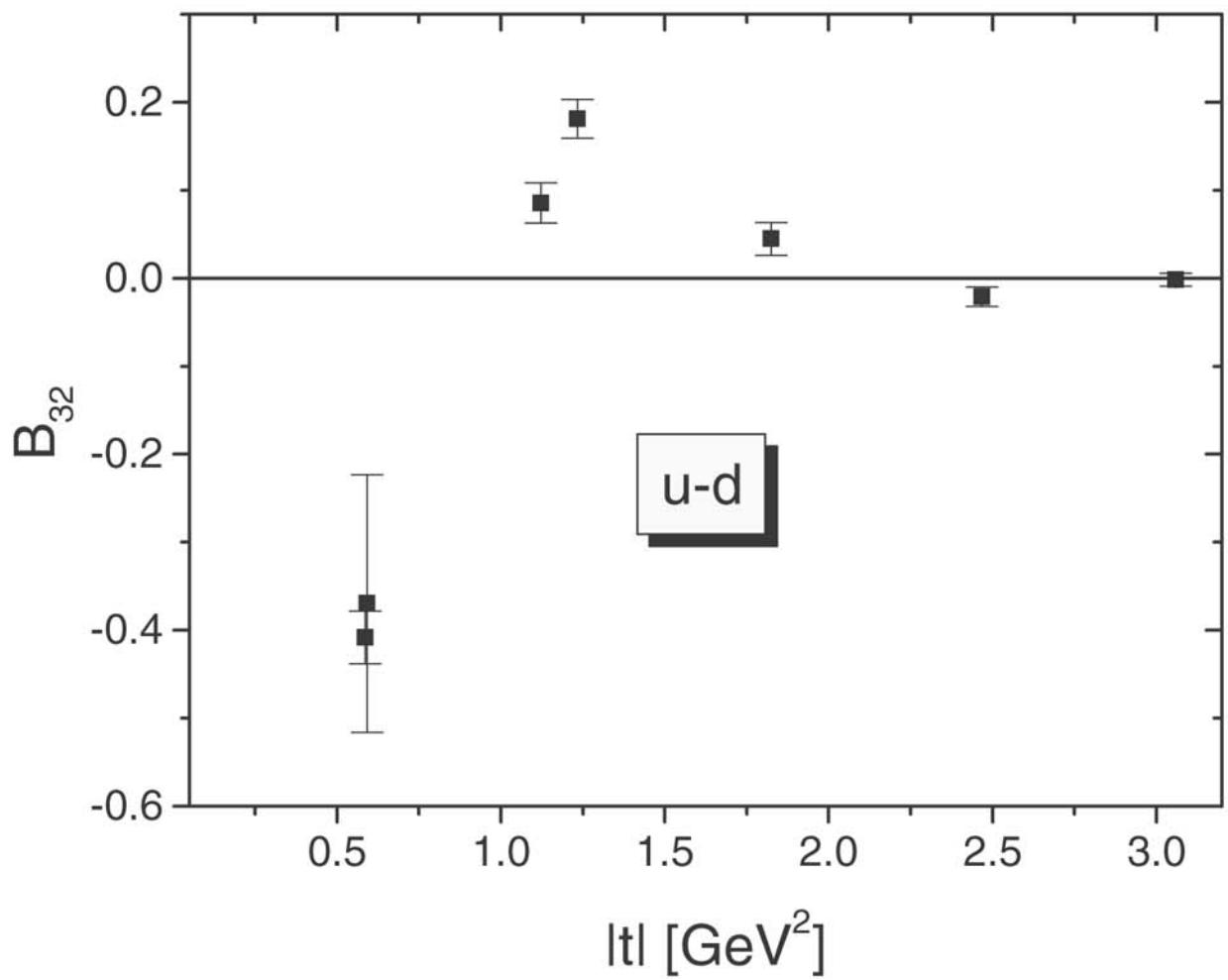












Hybrid Valence and Sea Quark Dirac Operators

Bär, Rupak, Shores hep-lat /0210050

- Conventional partially quenched chiral perturbation theory distinguishes m_s and m_v : Generalize to Dirac operator

$$S = S_{YM}[U] + S_W[\bar{\psi}_s, \psi_s, U] + S_{GW}[\bar{\psi}_v, \psi_v, U]$$

ψ_s : Grassman sea quarks

ψ_v : Grassman valence quarks + ghost quarks

- Chiral P.T. for $\mathcal{O}(a)$ corrections (to sea)
- NLO meson mass (flavor non-singlet)

$$\begin{aligned} M^2 &= m_v \\ &+ \frac{m_v}{48f^2\pi^2} [m_v - m_s - a + (2m_v - m_s - a) \ln(m_v)] \\ &- \frac{8m_v}{f^2} [(L_5 - 2L_8)m_v + 3(L_4 - 2L_6)m_s + 3(W_4 - W_6)a] \end{aligned}$$

- Goal: extend to staggered sea

Chiral Limit of Strongly Coupled U(N) Lattice Gauge Theories

Adams and Chandrasekharan

- Standard techniques unable to explore chiral limit
- Strong coupling U(n) with staggered quarks
 - Integrate out U analytically
 - Partition function: constrained sum of monomers on sites and dimers on links
 - Efficient cluster algorithm
 - Finite size scaling of chiral susceptibility yields precise chiral condensate in 3,4 d

$$\langle \bar{\psi} \psi \rangle = 0.2264(1) \quad U(1) \text{ d=4}$$

Research Plans - Hadronic Physics at Light Quark Masses

- Hadron structure

- Hybrid dynamical calculation

- Staggered sea quarks (MILC)**

- Domain wall valence quarks**

L/a	am_l	am_s	m_π MeV	m_π^2 GeV ²	Configs
20	0.1	0.1	609	.371	339
20	0.05	0.05	522	.272	414
20	0.03	0.05	448	.201	564
20	0.02	0.05	391	.153	385
20	0.01	0.05	304	.092	630
28	0.01	0.05	305	.093	165+

MILC Lattices with $a = 0.13\text{fm}$ to be used with valence domain wall fermions

20^3 $a = 0.13\text{ fm}$ $L = 2.6\text{ fm}$ $m_\pi = 300\text{ MeV}$

28^3 $a = 0.13\text{ fm}$ $L = 3.64\text{ fm}$ $m_\pi = 216\text{ MeV}$

Partially quenched $m_\pi^2 = 0.047, 0.093, 0.153$

- **Hadron structure production plans**
 - **Propagators:**
 - 1 Forward
 - 1 Backward pion
 - 4 Backward nucleon: 2 flavors \times 2 p
 - 6 color-spin components
 - $L_s = 24$
 - 500 Configurations
 - **Domain wall estimates:**
 - 5 masses on 20^2 : 69 GFyr
 - 3 masses on 28^2 : 113 GFyr
- **Disconnected diagrams with overlap fermions**
 - **Merge Kentucky, JLab, and Boston overlap effort in common QDP⁺⁺ code**
 - **Add flavor singlet matrix elements to existing non-singlet results**
 - **Estimate 98 GFyr**

- **Excited Baryon and meson spectrum at light quark masses**
 - **N ***
 - **Excited mesons**
 - **General set of operators for sources**
 - **Asymmetric lattices**
 - **Combine with needs of heavy-light hadron-hadron potential**
 - **Estimate 65 GFyr**